

GAM · Vector Calculations — Strategy & Tricks

Every vector question is a small table: fill the rows, work the columns, check the bound, move on.

Contents: The rules the passage assumes you know · Name the variant in five seconds · The components table · Shortcuts that are actually faster · The 90-second budget · Seven ways to get it wrong fast · Worked examples, easy to hard · Recognition drills · Guided walk-through: one pack, five questions · Progressive practice · Speed training · On exam day · Cheat sheet

The rules the passage assumes you know

A vector is a list of numbers with a direction attached: $(\mathbf{x}, \mathbf{y})$ in the plane, $(\mathbf{x}, \mathbf{y}, \mathbf{z})$ in space. dMAT passages spell out the vectors and restate the formula they want you to use — treat that restatement as a cue. The arithmetic is on you.

Exam facts About 60 questions in 90 minutes — roughly **90 seconds each** — one passage shared by four or five questions, four options, no calculator, **no negative marking**. A blank scores exactly like a wrong answer, so you always answer.

$\mathbf{a} + \mathbf{b} = (a_1+b_1, a_2+b_2, a_3+b_3)$ — add column by column

$\mathbf{a} - \mathbf{b} = (a_1-b_1, a_2-b_2, a_3-b_3)$ — from A to B is $\mathbf{B} - \mathbf{A}$

$k \cdot \mathbf{a} = (ka_1, ka_2, ka_3)$ — scaling; $k < 0$ flips the direction

$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$ — length; never the component sum

$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}||\mathbf{b}| \cos \theta$ — a number, not a vector

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin \theta$ — parallelogram area; $\mathbf{a} \times \mathbf{b} \perp$ both

$\hat{\mathbf{u}} = \mathbf{a}/|\mathbf{a}|$ — same direction, length 1

$\cos \theta = \mathbf{a} \cdot \mathbf{b} / (|\mathbf{a}||\mathbf{b}|)$

$\mathbf{a} \perp \mathbf{b} \Leftrightarrow \mathbf{a} \cdot \mathbf{b} = 0$; $\mathbf{a} \parallel \mathbf{b} \Leftrightarrow \mathbf{b} = k \cdot \mathbf{a}$ — one ratio, every component

The entire toolbox: nine lines. Everything else is arithmetic.

Two facts do most of the heavy lifting. First, the dot product is a **scalar**: if the stem asks for it, every vector option is dead before you compute. Second, both products are **bounded** — neither $|\mathbf{a} \cdot \mathbf{b}|$ nor $|\mathbf{a} \times \mathbf{b}|$ can exceed $|\mathbf{a}||\mathbf{b}|$, because $\cos$ and $\sin$ never exceed 1. No other check kills as many distractors.

The passage is the law If the passage defines the dot product component-wise, use components. If it hands you $|\mathbf{a}|$, $|\mathbf{b}|$ and θ , use $|\mathbf{a}||\mathbf{b}| \cos \theta$. Convert only when the question forces it — every conversion costs twenty seconds. Anything exotic will be defined in the passage; apply that definition as written.

Name the variant in five seconds

Before you touch a number, classify the question. Six variants cover almost everything, each with exactly one operation attached.

If the stem says...	Variant	Operation	Budget
net, resultant, total, ground velocity	Resultant	Add the columns (subtract for 'correction' or 'return')	20–30 s

If the stem says...	Variant	Operation	Budget
how far, length, distance, $ v $, speed	Magnitude	Square, add, root — look for a triple first	20–40 s
angle between, acute or obtuse, $\cos \theta$	Angle	Dot $\div a b $, then read $\cos$ from memory	45–60 s
work done, component along, projection	Projection / work	Dot with the direction (unit vector for ‘component along’)	40–60 s
perpendicular, orthogonal, normal to	Perpendicularity	Is the dot product zero?	15–25 s
doubled, halved, scaled by k , reversed, unit vector	Scaling	Multiply every component by k ; magnitude scales by $ k $	10–20 s

Two extra labels: **area**, **torque**, ‘**perpendicular to both**’ means cross product, usually just $|a||b| \sin \theta$; ‘**find k such that...**’ means turning the stated condition into one equation. If a stem fits no row, re-read the passage — it has defined the operation for you.

The components table

Every question here is solved the same way: write the vectors as **rows**, one column per axis, and work **down the columns**. It looks childish. It is also why you stop dropping minus signs.

Row	x	y	z
r	7	–2	0
u	–3	5	0
$r + u$	4	3	0
$r - u$	10	–7	0
$r \cdot u$ (products)	–21	–10	0

Sum the products row: $r \cdot u = -31$, negative, so the angle between r and u is obtuse. $|r + u| = \sqrt{(16 + 9)} = 5$, a 3-4-5 triple — no squaring needed. One table, four answers.

1. **Extract.** Copy every vector from the passage into the table before reading the questions. One passage feeds four or five of them.
2. **Label the operation** from the stem and write it as a new row.
3. **Work the columns** one at a time — sign first, then size.
4. **Collapse.** Magnitude squares-and-adds: $|a| = \sqrt{x^2 + y^2 + z^2}$. Dot multiplies-and-adds: $a \cdot b = a_x b_x + a_y b_y + a_z b_z$, and $\cos \theta = a \cdot b \div (|a||b|)$. Each turns a row into one number.
5. **Bound check.** At most $|a||b|$? At least the largest component? Then pick the option.

One table per passage, not per question The next question almost always reuses a row you already have: $r + u$, then $|r + u|$, then the correction back. Write the rows once and later questions are ten-second lookups.

Shortcuts that are actually faster

- **Pythagorean triples, memorised:** 3-4-5, 5-12-13, 8-15-17, 7-24-25 and multiples (6-8-10, 9-12-15). In 3D: $(1, 2, 2) \rightarrow 3$, $(2, 3, 6) \rightarrow 7$, $(1, 4, 8) \rightarrow 9$, $(2, 6, 9) \rightarrow 11$.
- **The sign of the dot product is free information:** positive $\rightarrow$ acute, zero $\rightarrow$ right angle, negative $\rightarrow$ obtuse. Half the angle questions ask no more.
- **Cos from memory:** $0^\circ \rightarrow 1$, $30^\circ \rightarrow \sqrt{3}/2$, $45^\circ \rightarrow 1/\sqrt{2}$, $60^\circ \rightarrow 1/2$, $90^\circ \rightarrow 0$; sin is the same list backwards. Read it in reverse too: $\cos \theta = 1/2$ means 60° .
- **Bound before you compute:** $|a \cdot b|$ and $|a \times b|$ cannot exceed $|a||b|$. With $|a| = 4$ and $|b| = 5$, any option above 20 is dead.
- **A magnitude is never smaller than its biggest component:** $|(3, k, 6)|$ cannot be 5.
- **Square roots multiply cleanly:** $\sqrt{2} \cdot \sqrt{2} = 2$, $\sqrt{3} \cdot \sqrt{12} = 6$. Keep them symbolic; approximating $\sqrt{2} \approx 1.41$ mid-way loses the cancellation.
- **Parallel by ratio, not by angle:** $(2, -4, 6)$ and $(-1, 2, -3)$? Every component is -2 times the other — parallel and opposite. No cos needed.
- **Unit vectors usually hide a triple:** $(0.6, 0.8)$, $(1/3)(1, 2, 2)$. If a 'unit' vector isn't unit, make one: divide by the magnitude.

The 90-second budget

Ninety seconds a question, but a vector pack is four or five questions on one passage — think **six or seven minutes for the pack**. Spend the first 40 seconds on the table and each question then costs 20–60 seconds.

1. **0–10 s:** read the passage once, marking every vector and formula it states.
2. **10–40 s:** fill the components table, plus a spare row for the obvious next operation.
3. **Per question:** 5 s to classify the variant, 10 s to bound-check the options, the rest computing down the columns, 5 s to sanity-check — then commit.

Where the seconds actually go Not into arithmetic. Into re-reading the passage a third time, and squaring numbers you never needed to because $(6, 8)$ was sitting right there.

Seven ways to get it wrong fast

1. **Wrong subtraction order.** 'From A to B' is $B - A$; the correction returning from r' to r is $r - r' = -u$. An answer with every sign flipped means you subtracted backwards — and that option is always on the list.
2. **Forgetting the square root.** $6^2 + 8^2 = 100$ is not a length. Watch for '100 m' next to '10 m'.
3. **The component sum posing as a magnitude.** $(6, 8)$ is not 14 long, yet $x + y$ appears in nearly every magnitude question.
4. **A product bigger than $|a||b|$.** $\sin \theta \leq 1$ and $\cos \theta \leq 1$, so neither $|a \times b|$ nor $|a \cdot b|$ can exceed $|a||b|$.
5. **Adding magnitudes instead of vectors.** $|(9, 12)| + |(-3, -4)| = 20$, but the sum $(6, 8)$ is only 10 long. Lengths add only when both vectors point the same way.
6. **Degrees and radians.** $\pi/3$ is 60° , $\cos = 1/2$. Read the unit before you read the table.
7. **Sin for cos.** Dot uses cos (the 'along' part), cross uses sin (the 'across' part). The 30° torque question offers $\sqrt{3}/2$ as bait.

The universal antidote After every answer, ask: right order? root taken? below the bound? Five seconds, and most careless errors die there.

Worked examples, easy to hard

Each example uses a mini passage in the exam's style. Cover the steps, attempt it inside the time target, then compare.

Net force and its magnitude EASY · target 45s

What is the magnitude of the net force N?

Two harbour tugs push a barge. Along two perpendicular horizontal axes, the first applies $P = (9, 12)$ kN and the second $Q = (-3, -4)$ kN. The net force is $N = P + Q$, and the magnitude of (a, b) is $\sqrt{a^2 + b^2}$.

- A. 10 kN
- B. 14 kN
- C. 20 kN
- D. 100 kN

Steps

1. Table: $x \rightarrow 9 - 3 = 6$; $y \rightarrow 12 - 4 = 8$. So $N = (6, 8)$.
2. Spot the triple: $(6, 8)$ is $2 \times (3, 4)$, so $|N| = 10$. No squaring needed.
3. Distractors: 14 is the component sum, 100 forgot the root, 20 is $|P| + |Q| = 15 + 5$. Lengths add only when both point the *same* way; Q points backwards, so they subtract: $15 - 5 = 10$, matching the columns.

Answer: 10 kN

Perpendicular to a displacement EASY · target 50s

Which vector is perpendicular to d?

A robot arm's tip moves from $A = (2, 5, -1)$ to $B = (4, 1, 3)$, in centimetres, so the displacement is $d = B - A$. The dot product of (a_1, a_2, a_3) and (b_1, b_2, b_3) is $a_1b_1 + a_2b_2 + a_3b_3$, and two vectors are perpendicular exactly when it is zero.

- A. $(2, 4, 4)$
- B. $(2, 1, 0)$
- C. $(1, 1, 1)$
- D. $(-2, 4, -4)$

Steps

1. Order first: $d = B - A = (4 - 2, 1 - 5, 3 - (-1)) = (2, -4, 4)$. Mind the double negative in z.
2. Products row per option: $(2, 1, 0) \rightarrow 4 - 4 + 0 = 0$. That is the one.
3. Check the rest: $(2, 4, 4) \rightarrow 4 - 16 + 16 = 4$; $(1, 1, 1) \rightarrow 2$; $(-2, 4, -4)$ is $-d$ — backwards is 180° , not 90° .

Answer: $(2, 1, 0)$

Angle between two bonds MEDIUM · target 60s

What is the angle between the two bonds?

In a crystal model, two bonds leave the same atom along $u = (1, 1, 0)$ and $v = (1, 0, 1)$. For any two vectors, $u \cdot v = u_1v_1 + u_2v_2 + u_3v_3$, and also $u \cdot v = |u||v| \cos \theta$, where θ is the angle between them. $|(x, y, z)| = \sqrt{x^2 + y^2 + z^2}$.

- A. 30°
- B. 45°
- C. 60°
- D. 90°

Steps

1. Products row: $1 \cdot 1 + 1 \cdot 0 + 0 \cdot 1 = 1$. Positive, so acute — 90° is gone.
2. Magnitudes: $|u| = |v| = \sqrt{2}$. Keep them symbolic: $|u||v| = \sqrt{2} \cdot \sqrt{2} = 2$.
3. $\cos \theta = 1/2$, so $\theta = 60^\circ$. 45° baits 'diagonal of a square'; 30° baits reading cos as sin.

Answer: 60°

Torque with a bound check MEDIUM · target 50s

What is the magnitude of the turning effect?

A technician pushes a valve handle. The handle's position vector r from the pivot has magnitude $|r| = 0.5$ m, the push F has magnitude $|F| = 20$ N, and the angle between r and F is 30° . The turning effect is $r \times F$, with magnitude $|r||F| \sin \theta$.

- A. $5 \text{ N}\cdot\text{m}$
- B. $5\sqrt{3} \text{ N}\cdot\text{m}$
- C. $10 \text{ N}\cdot\text{m}$
- D. $40 \text{ N}\cdot\text{m}$

Steps

1. Bound first: $|r||F| = 0.5 \times 20 = 10$. Anything above 10 is impossible — 40 dies at once, and 10 itself would need $\theta = 90^\circ$.
2. Cross product means sin, not cos: $\sin 30^\circ = 1/2$. The $5\sqrt{3}$ option is what $\cos 30^\circ = \sqrt{3}/2$ produces.
3. $10 \times 1/2 = 5 \text{ N}\cdot\text{m}$. Sanity: a push mostly along the handle turns it weakly — half the maximum fits.

Answer: $5 \text{ N}\cdot\text{m}$

Work along a ramp HARD · target 75s

How much work does the rope do?

A crate is hauled 5 m up a ramp whose direction is the unit vector $\hat{u} = (0.6, 0.8)$, so the displacement is $d = 5\hat{u}$. The rope applies a constant force $F = (24, 7)$ N. Work done is $W = F \cdot d$ with the dot product taken component-wise; the scalar projection of F onto $\hat{u}$ is $F \cdot \hat{u}$. The magnitude of (a, b) is $\sqrt{a^2 + b^2}$.

- A. 20 J
- B. 100 J
- C. 117 J
- D. 125 J

Steps

1. Make d concrete: $5 \times (0.6, 0.8) = (3, 4)$; $|d| = 5$ as promised.
2. Bound: $|F| = 25$ ($7-24-25$), so $|F||d| = 125$ — but that needs F parallel to d , and $24/3 \neq 7/4$. Out.
3. Columns: $24 \times 3 = 72$; $7 \times 4 = 28$; $W = 100$ J.
4. The rest: 20 is $F \cdot \hat{u}$, the projection in newtons, still missing the 5 m; 117 crosses the columns ($24 \times 4 + 7 \times 3$).

Answer: 100 J

Unknown component under two conditions HARD · target 90s

What is the value of k ?

A stage-rigging cable runs along $w = (3, k, 6)$ in metres, with its length fixed at $|w| = 7$ m. It must also make an obtuse angle with the reference direction $c = (1, 2, -1)$. Recall $|(x, y, z)| = \sqrt{x^2 + y^2 + z^2}$ and $w \cdot c = w_1c_1 + w_2c_2 + w_3c_3 = |w||c| \cos \theta$, so $w \cdot c$ has the sign of $\cos \theta$.

- A. -4
- B. -2
- C. 2
- D. ± 2

Steps

1. Length: $9 + k^2 + 36 = 49$, so $k^2 = 4$ and $k = \pm 2$. Don't stop — ' ± 2 ' is the option for people who do.
2. Obtuse means $\cos \theta < 0$, so $w \cdot c < 0$. Products row: $3 + 2k - 6 = 2k - 3$.
3. $k = 2$ gives $+1$ (acute, rejected); $k = -2$ gives -7 (obtuse). So $k = -2$.
4. Verify: $w = (3, -2, 6)$, $|w| = \sqrt{9 + 4 + 36} = 7$. The -4 option skips the root on $k^2 = 4$ and borrows the minus from 'obtuse'.

Answer: -2

Recognition drills

Say the operation out loud, then the answer. No paper on the first pass — the point is to make classification reflexive.

Which operation?

Read the stem, name the variant, answer inside the time.

1. A walker goes (3, 4) km, then (-1, 2) km. Net displacement? (12s)

Answer: Resultant → add the columns: (2, 6) km

2. How far from its start is a drone at (6, 8) m? (8s)

Answer: Magnitude → 3-4-5 doubled: 10 m

3. Are (2, 1) and (1, -2) at right angles? (10s)

Answer: Dot product → $2 - 2 = 0$: yes

4. A rod along (1, -2, 2) is stretched to three times its length. New vector? (12s)

Answer: Scalar multiple → (3, -6, 6); length 3 → 9

5. Work done by $F = (5, 2)$ N over $d = (2, 3)$ m? (12s)

Answer: Dot product → $10 + 6 = 16$ J

6. Area of the parallelogram spanned by lengths 3 and 4 at 90° ? (10s)

Answer: Cross-product magnitude → $3 \times 4 \times \sin 90^\circ = 12$

7. Is (2, -4, 6) parallel to (-1, 2, -3)? (12s)

Answer: Ratio test → each component is -2 times the other: parallel, opposite

Numbers from memory

Magnitudes, dot products and trig values without squaring on paper.

1. $|(5, 12)|$ (6s)

Answer: 13 (5-12-13)

2. $|(2, -1, 2)|$ (8s)

Answer: 3 ($4 + 1 + 4 = 9$)

3. $(3, -2) \cdot (4, 6)$ (10s)

Answer: $12 - 12 = 0$ → perpendicular

4. $|a| = 6$, $|b| = 5$, $\theta = 30^\circ$: $|a \times b|$ (10s)

Answer: $6 \times 5 \times 1/2 = 15$

5. $r = (7, -2)$, $u = (-3, 5)$: $r - u$ (10s)

Answer: (10, -7) — not (-10, 7)

6. Unit vector along (3, 4) (8s)

Answer: (0.6, 0.8): divide by 5

Bound check

Using only magnitudes, decide instantly whether the option is possible.

1. $|a| = 4$, $|b| = 5$. Could $a \cdot b = 24$? (6s)

Answer: No — the bound is 20

2. $|a| = 4$, $|b| = 5$. Could $|a \times b| = 20$? (6s)

Answer: Yes, only at $\theta = 90^\circ$

3. $w = (3, k, 6)$. Could $|w| = 5$? (8s)

Answer: No — shorter than its own component 6

4. $a \cdot b = -8$. Acute or obtuse? (6s)

Answer: Obtuse — negative

5. 10 m, 14 m, 100 m for $|(6, 8)|$: which two are traps? (8s)

Answer: 14 (component sum), 100 (no root)

6. $|a + b|$ with $|a| = 3$, $|b| = 4$, $\theta = 90^\circ$? (8s)

Answer: 5 by Pythagoras; 7 only if they point the same way

Guided walk-through: one pack, five questions

Here is a full pack the way it lands in the exam, with the thinking that should run at each moment.

A warehouse robot has moved $r = (7, -2)$ m from its dock. Its next scheduled move is $u = (-3, 5)$ m, giving $r' = r + u$. A correction c may be added afterwards. $|(x, y)| = \sqrt{x^2 + y^2}$, and for two vectors $a \cdot b = a_1b_1 + a_2b_2 = |a||b| \cos \theta$.

One passage, five linked questions — the standard pack format.

- Seconds 0–30, the table.** Rows $r = (7, -2)$ and $u = (-3, 5)$. Pre-compute $r + u = (4, 3)$ and, because 'correction' is in the passage, $-u = (3, -5)$.
- Q1: r' ?** Lookup: $(4, 3)$. Five seconds.
- Q2: the correction returning the robot from r' to r ?** $r - r' = -u = (3, -5)$. The $(-3, 5)$ option is the wrong-order trap. Ten seconds.
- Q3: final displacement if $c = (2, -1)$?** $r' + c = (6, 2)$. Ten seconds.
- Q4: $|r'|$?** $(4, 3) \rightarrow 5$ m on sight. A $\sqrt{\text{something}}$ option means the wrong row got squared: $|r| = \sqrt{53}$, $|u| = \sqrt{34}$. Five seconds.
- Q5: angle between r' and u , acute or obtuse?** $4 \cdot (-3) + 3 \cdot 5 = 3 > 0$, so acute. Fifteen seconds.

Five questions, about 75 seconds of computing, most of it in the table. Front-load, then harvest.

Glance at the options first In linked questions, read the options before computing. If three are vectors and one is a number, or the reverse, the stem's operation says which kind can survive — a free elimination.

Progressive practice

Untimed. The goal is zero careless errors, not speed. After each set, sort every mistake into one of the seven traps — the sorting is the practice.

Live drill online: Easy · resultants, magnitudes, perpendicularity —

<https://dmatprepguide.com/?try=drill&cat;=gam&tier;=easy&n;=6&timed;=0>

Live drill online: Medium · angles, cross-product magnitudes, unknown components —

<https://dmatprepguide.com/?try=drill&cat;=gam&tier;=medium&n;=6&timed;=0>

Live drill online: Hard · projections, two-condition constraints, 3D packs —

<https://dmatprepguide.com/?try=drill&cat;=gam&tier;=hard&n;=6&timed;=0>

Move up a tier when you score five of six with the three-question check applied.

Speed training

Now the clock: three rounds, the target dropping each time. Write the table every round, even when you think you don't need it — the habit is what gets faster, not your arithmetic.

1. **Round 1 — 75 s per question.** Table, classify, bound check, compute, one written sanity check.
2. **Round 2 — 60 s.** Same routine, sanity check in your head.
3. **Round 3 — 45 s.** Skip the squaring whenever a triple is visible; guess and move at 45 s.

Live drill online: Timed · medium, 60 s each —

<https://dmatprepguide.com/?try=drill&cat=gam&tier=medium&n=6&timed=1>

Live drill online: Timed · hard, 75 s each —

<https://dmatprepguide.com/?try=drill&cat=gam&tier=hard&n=6&timed=1>

The time target is a ceiling At 90 seconds and not converging? Take the option that survives the bound and sign checks, and move. Vector packs are where you gain time for the rest of the GAM, not where you spend it.

On exam day

Vector packs arrive as one passage plus four or five linked questions — the most predictable points in the GAM, if you run the routine.

- Read the passage once; build the table before question 1.
- Copy the passage's formula exactly — it decides components versus $|a||b| \cos \theta$.
- Classify each stem in five seconds.
- Bound-check the options before computing.
- Subtraction order: 'from A to B' is $B - A$.
- Magnitude: root taken? At least the largest component?
- Sign of the dot product → acute or obtuse; sin for cross, cos for dot.
- Never leave a blank.

Live drill online: Exam simulation · hard passage questions, 90 s each —

<https://dmatprepguide.com/?try=drill&cat=gam&tier=hard&n=6&timed=1>

Linked questions share a passage, not their answers. If Q3 feels wrong, don't rebuild Q1 — re-check the single row you used.

Cheat sheet

Add / subtract column by column. From A to B = $B - A$.

Scale: $k \cdot (x, y, z) = (kx, ky, kz)$; $|k \cdot a| = |k||a|$.

Magnitude $\sqrt{x^2 + y^2 + z^2}$: take the root; never below the largest component.

Triples: 3-4-5, 5-12-13, 8-15-17, 7-24-25; 3D $(1,2,2) \rightarrow 3$, $(2,3,6) \rightarrow 7$, $(1,4,8) \rightarrow 9$.

Dot: $a \cdot b = \sum a_i b_i = |a||b| \cos \theta$ — a number; $|a \cdot b| \leq |a||b|$.

Angle: $\cos \theta = a \cdot b / (|a||b|)$; sign → acute (+), right (0), obtuse (-).

$\cos 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ = 1, \sqrt{3}/2, 1/\sqrt{2}, 1/2, 0$; sin is the list backwards.

Perpendicular $\Leftrightarrow a \cdot b = 0$. Parallel $\Leftrightarrow b = k \cdot a$.

Cross: $|a \times b| = |a||b| \sin \theta \leq |a||b|$; $\perp$ both; parallelogram area.

Unit vector $\hat{u} = a/|a|$. Component along $\hat{u} = a \cdot \hat{u}$. Work = $F \cdot d$.

Ten lines to reread the night before.

- Table built before Q1
- Operation named from the stem
- Impossible options crossed out
- Order, root, bound checked

■ Answered — never blank

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